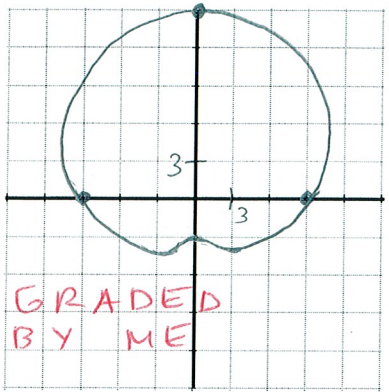
Consider the graph of the polar equation $r = -9 + 6\sin\theta$. $\left|\frac{-9}{6}\right| = \frac{3}{2} > 1$ **SCORE: ____ / 6 PTS**



[a] Fill in the blanks.

[i] The shape of the graph is a/an (1) MUST HAVE "DIMPLE" TO EARN ANY POINTS LIMAÇON WITH A DIMPLE (1/2). **limaçon**

[ii] The graph (1/2) DOES NOT **does / does not** pass through the pole.

[iii] Find the **rectangular coordinates** of the

θ	r
0	-9
$\frac{\pi}{2}$	-3
π	-9
$\frac{3\pi}{2}$	-15

$x-$ intercept(s)

$x =$ (-9, 9) (1) **(rectangular coordinates)**

$y-$ intercept(s)

$y =$ (-3, 15) (1) **(rectangular coordinates)**

[b] Sketch the graph on the grid provided above. **You must provide a scale for the axes & plot all points from part [a][iii] above.**

The graph of the polar equation $r = 2 + 4\sin 3\theta$ is symmetric only over $\theta = \frac{\pi}{2}$. **SCORE: ____ / 5 PTS**

[a] What is the minimum interval of the graph you need to plot first, before using reflections to draw the rest of the graph ?

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (1/2)

[b] **Algebraically** find the angles in the minimum interval in part [a] at which the graph goes through the pole.
NOTE: You will NOT receive credit for just plugging in numbers to guess the answers.

$2 + 4\sin 3\theta = 0$ (1)

$\sin 3\theta = -\frac{1}{2}$ (1/2)

$3\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}$ (1 1/2)

$\theta = -\frac{5\pi}{18}, -\frac{\pi}{18}, -\frac{7\pi}{18}$ (1 1/2)

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$-\frac{3\pi}{2} \le 3\theta \le \frac{3\pi}{2}$

Convert the polar equation $r^2 = \sin 2\theta$ to rectangular, and simplify as shown in the website handout. **SCORE: _____ / 4 PTS**

NOTE: Your final answer should NOT have fractions, radicals, nor negative or fractional exponents.

$r^2 = 2\sin\theta\cos\theta$ (1)

$r^2 = 2\frac{y}{r}\frac{x}{r}$ (1) ← ALTERNATELY: $r^2 \cdot r \cdot r = 2(r\sin\theta)(r\cos\theta)$

$r^4 = 2xy$

$(x^2+y^2)^2 = 2xy$ (2)

Consider the graph of the polar equation $r = 5 - 2\sin 3\theta$.

SCORE: ____ / 5 PTS

NOTE: The $(-r, \theta)$, $(-r, \pi-\theta)$ and $(r, \pi+\theta)$ tests do NOT show that the graph is symmetric

POLE POLAR AXIS POLE

Using the information above, and the tests and shortcuts shown in lecture, test if the graph is symmetric over the pole, the polar axis, and/or $\theta = \frac{\pi}{2}$. State your conclusions in the table. **NOTE: Run as FEW tests as needed to prove your answers are correct.**

POLAR AXIS

$(r, -\theta)$

$r = 5 - 2\sin 3(-\theta)$

(1) $r = 5 - 2\sin(-3\theta)$

(½) $r = 5 + 2\sin 3\theta$

$\theta = \frac{\pi}{2}$

$(r, \pi-\theta)$

$r = 5 - 2\sin 3(\pi-\theta)$

$r = 5 - 2\sin(3\pi - 3\theta)$ (1)

$r = 5 - 2[\sin 3\pi \cos 3\theta - \cos 3\pi \sin 3\theta]$

$r = 5 - 2\sin 3\theta$ (1)

Type of symmetry	Conclusion
Over the polar axis	NO CONCLUSION
Over $\theta = \frac{\pi}{2}$	SYMMETRIC
Over the pole	NO CONCLUSION

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Fill in the blanks. **NOTE: All answers for parts [a] and [c] must be positive.**

$r = \sqrt{63+21} = \sqrt{84}$

$\ominus = \pi + \tan^{-1}\frac{\sqrt{21}}{3\sqrt{7}} = \pi + \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = \pi - \frac{\pi}{6}$

SCORE: _____ / 4 PTS

[a] The point with rectangular co-ordinates $(-3\sqrt{7}, \sqrt{21})$ has polar co-ordinates ($2\sqrt{21}$, $\frac{5\pi}{6}$). (½) (½)

[b] The point with polar co-ordinates $(14, -\frac{5\pi}{6})$ has rectangular co-ordinates ($-7\sqrt{3}$, -7). (1)

$x = 14\cos\left(-\frac{5\pi}{6}\right) = 14 \cdot -\frac{\sqrt{3}}{2}$

$y = 14\sin\left(-\frac{5\pi}{6}\right) = 14 \cdot -\frac{1}{2}$

[c] The polar co-ordinates $(-12, -\frac{13\pi}{9})$ correspond to the same point as the polar co-ordinates $(12,$ $\frac{14\pi}{9}$ $)$ (1) and $(-12,$ $\frac{5\pi}{9}$ $)$. (½)

$-\frac{13\pi}{9} + \pi + 2\pi$ $\quad$ $-\frac{13\pi}{9} + 2\pi$

POLAR AXIS

The $(r, -\theta)$ test is the only test that indicates that the graph of $r = 4 - 2\cos 3\theta$ is symmetric. **SCORE: _____ / 6 PTS**

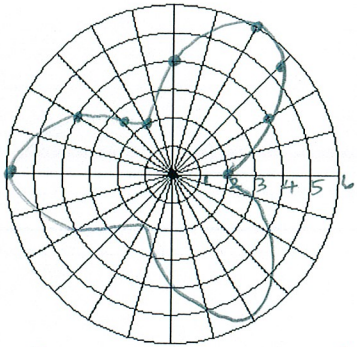
The graph passes through the polar points $(2, 0), (4, \frac{\pi}{6}), (5.4, \frac{\pi}{4}), (6, \frac{\pi}{3})$ and $(4, \frac{\pi}{2})$, and does **NOT** pass through the pole.

[a] What is the minimum interval of the graph you need to plot first, before using reflections to draw the rest of the graph ?

$[0, \pi]$ (1)

[b] Find the value of r (**rounded to 1 decimal place**) for all other common angles in the minimum interval in part [a].
You do **NOT** need to find r for the angles given in the original problem description.
NOTE: You do NOT need to show work, only answers.

θ	r
$\frac{2\pi}{3}$	$4-2\cos 2\pi = 4-2\cdot 1 = 2$ (½)
$\frac{3\pi}{4}$	$4-2\cos\frac{9\pi}{4} = 4-2\cdot\frac{\sqrt{2}}{2} = 4-\sqrt{2} \approx 2.6$ (½)
$\frac{5\pi}{6}$	$4-2\cos\frac{5\pi}{2} = 4-2\cdot 0 = 4$ (½)
π	$4-2\cos 3\pi = 4-2\cdot -1 = 6$ (½)



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[c] Sketch the graph on the grid provided above. **You must provide a scale for the polar axis & plot all points given and found.**